# Optimal value for the exponential term of Wilmink's function according to current Holstein lactation curves in Japan

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**Abstract**

We compared values of Wilmink's exponential term to describe the lactation curves of Holstein cows in Japan. Data were a total of 100,971,798 test-day records from the first through fifth parities during 1991 through 2018. The lactation curve model used fourth-order Legendre polynomials and Wilmink's exponential term. In total, 810 analyses were executed to compare six values (–0.02, –0.03, …, –0.07) for the exponential term to select the one that yielded the smallest root mean square error. For all parities, daily milk yield and lactation persistency increased consistently and peak lactation days occurred later from year to year. For the years evaluated, the optimal exponential term was –0.05 for first and second parities, –0.04 for third parity, and –0.03 for fourth and fifth parities. The change in the exponential parameter with increasing year was related to delays in peak lactation.

Key words : Dairy cattle, Holstein, Lactation curve, Wilmink's function, Lactation persistency

## Introduction

Lactation curves have been highly studied because of their usefulness in the genetic analysis of test-day records for assessing effects on lactation period (Ptak and Schaeffer, 1993). In addition, lactation curves are used to estimate milk yields. Lactation curve models—including those from Wood (Wood, 1967), Ali–Schaeffer (Ali and Schaeffer, 1987), and Wilmink (Wilmink, 1987)—vary widely, and their respective strengths and weaknesses have been compared (Druet et al., 2003; Silvestre et al., 2006). In this regard, De Melo et al. (2007) proposed a model that combines the Legendre polynomial with Wilmink's exponential function.

Although previous lactation curve models typically were selected according to their accuracy in estimating 305-day milk yield (a widely used indicator of total milk yield during the lactation period), recent efforts in genetic improvement have focused on both 305-day milk yield and the shape of the lactation curve, as represented by lactation persistency, which describes the decline in milk yield after peak lactation (Muir et al., 2004). Therefore, it is important to have a lactation curve model that effectively represents the overall pattern of lactation.

Since 2010, a random regression repeatability test-day model has been used for national genetic evaluation of dairy cows in Japan. The model involves fourth-order Legendre polynomials with the exponential term of –0.05 for Wilmink's function (Hagiya, 2019). Wilmink's exponential function is related to the time from early to peak lactation, and a value of –0.05 typically is used (Wilmink, 1987). Although Yamaguchi et al (2007) evaluated the degree of Legendre polynomials, but they did not determine the optimal exponential term for Wilmink's function.

Using test-day yields recorded in Japan during 2007 through 2009, Sasaki et al (2013) determined the optimal exponential term for the lactation curves for each parity. However, very few studies in Japan have focused on changes in the lactation curve with calving year. In that regard, the optimal value for the exponential term in Wilmink's model might differ depending on the cow population, parity, or calving year. Knowing the optimal value for Wilmink's exponential term might yield more accurate estimates of the lactation curve shape and lactation persistency.

The objective of the current study was to estimate the most appropriate value for Wilmink's exponential term to optimally describe the lactation curves of Holstein cows in Japan.

## Material and Methods

### Data

The data were obtained from the Livestock Improvement Association of Japan (Tokyo, Japan) and comprised 100,971,798 test-day records from the first through fifth lactations during January 1991 through November 2018. Test-day yields on days in milk (DIM) from 6 to 305 days were used in the analyses. Peak yields were estimated by using simple moving averages of the 7 days before and after the test day, to smooth partial changes for graphical representation. Lactation persistency was estimated as the difference between the milk yields at 240 DIM and 60 DIM, as used in dairy sire and cow evaluations in Japan (Yamazaki et al, 2013).

**Statistical analysis**

The lactation curve model was based on fourth-order Legendre polynomials and the exponential term of Wilmink's function, as used for national genetic evaluation in Japan (Hagiya, 2019):

$$y_{ijt} = HTD_i + a_0 + a_1 L_1(x_t) + a_2 L_2(x_t) + a_3 L_3(x_t) + a_4 L_4(x_t) + a_5 e^{wt} + e_{ijt} \ ,$$

where $y_{ijt}$ is the milk yield on the test day, $HTD_i$ is the fixed effect of herd and test day, $a_0$ through $a_5$ are coefficients for each term, $L_1$ through $L_4$ are the terms for the Legendre polynomials, $x_t$ is the standardized DIM on test day $t$, $wt$ is the exponential term of Wilmink's function $w$ multiplied by test day $t$, $e^{wt}$ and $e_{ijt}$ accounts for random residuals. Standardized DIM was obtained as:

$$x_t = 2 \times (t - t_{min})/(t_{max} - t_{min}) - 1.$$

The values for Wilmink's exponential term were selected to yield the smallest root mean squared error. Random residual values were estimated by using the BLUPF90 program (Misztal et al., 2002). In total, 810 analyses were executed to compare six values (–0.02, –0.03, … –0.07) for Wilmink's exponential term $w$ within 27 test years and 5 parities.

We defined lactation persistency as the difference between milk yields at 240 and 60 DIM (Yamazaki et al, 2013). In the current study, lactation persistency was defined as follows:

$$Lactation\ pesistency = M_{60DIM} - M_{240DIM} + 100\ ,$$

where $M_{60DIM}$ and $M_{240DIM}$ are test-day milk yields at 60 DIM and 240 DIM, respectively.

## Results and Discussion

### *Change in lactation curve with test year*

The number of records, daily milk yields, standard deviations, and minimum and maximum yields for each parity are given in Table 1. Overall, daily milk yield increased from 26.1 kg for the first lactation to 31.5 kg for the third lactation. From 1991 through 2018, the average daily milk yield increased from 22.3 to 28.6 kg for the first lactation, 25.9 to 33.0 kg for the second, 27.4 to 34.1 kg for the third, 27.8 to 33.9 kg for the fourth, and 27.8 to 33.3 kg for the fifth lactation (Figure 1). Therefore, daily milk yield increased consistently from year to year for all parities.

Trends in the lactation curve according to test year and parity are shown in Figure 2. The shapes of the lactation curves for the fourth and fifth lactations were almost the same as for the third lactation. Overall milk production increased year by year, particularly during later lactation periods. Compared with those for later parities, peak yield was lower, and the subsequent decrease in milk yield was slower, for first parity, consistent with the higher lactation persistency of first parity compared with later parities.

The peak milk-yield day was around 35 days after calving for all parities in 1991, compared with 55 days after calving for first lactation, 38 days after calving for second lactation, and 40 days after calving after second lactation in 2018; that is, for all parities, the peak day was noticeably later in 2018 than 1991, with the latest peak in 2018 for first lactation (Figure 3). From 1991 to 2018, lactation persistency increased from 93 to 95 for first lactation, from 88 to 89 for second lactation, from 85 to 88 for third lactation, from 86 to 88 for fourth lactation, and from 85 to 89 for fifth lactation. Lactation persistency tended to improve with increasing test year and was particularly high for the first lactation (Figure 4), similar to results from a previous study (Mahdi at el, 2019). The association between delayed peak lactation and increased lactation persistency is consistent with previous reports (Muir et al., 2004). The decrease in lactation persistency with increasing parity reflects the moderate to large positive correlation between peak milk - yield day and persistency (Mahdi et al., 2019). Milk yield in early lactation might result somewhat lower lactation persistency, because of the negative genetic correlation (–0.09) between these traits (Farhangfar and Rowlinson, 2007). Milk yield in early lactation was lowest in the first lactation and increased during later lactations (Figure 2). This effect may account for lactation persistency being highest during the first lactation but somewhat lower during later lactations (Figure 4). However, Yamazaki et al. (2013) reported that the genetic correlations between 305-day milk yield and lactation persistency were positive (range, 0.11 to 0.36). Therefore, improvements in 305-day milk yield and lactation persistency have resulted in a later peak day.

***Optimal values for Wilmink's exponential term***

The optimal values for Wilmink's exponential term for each parity are shown in Table 2; the root mean squared errors for each value of Wilmink's exponential term are shown in Appendices 1 through 5. The optimal values for

Wilmink's exponential term varied from –0.05 to –0.07 for first lactation during 1991 through 1999, but –0.05 emerged as best for most test years thereafter. For most test years, the optimal value for the Wilmink exponential term was –0.05 during the second parity and –0.04 for the third parity. For the fourth parity, the optimal Wilmink's exponential parameter was –0.04 before 2003 but –0.03 thereafter, and for the fifth parity the optimal value was –0.04 from 1991 through 1993 but –0.03 for most subsequent years. Except in the case of third parity, the absolute value of the optimal Wilmink's exponential term tended to decrease with year.

Changes in exponential parameters with increasing year are suggested to reflect delays in peak lactation. Sasaki et al. (2013) used a nonlinear least squares method based on lactation curves generated from fourth-order Legendre polynomials and Wilmink's exponential term to estimate optimal values for Wilmink's exponential term: –0.07 for first parity, –0.05 for second, –0.04 for third, and –0.05 for fourth parity. As in our current study, the previously calculated optimal exponential terms (Sasaki et al., 2013) were greater for the second and later parities than for the first parity.

For genetic evaluation of dairy cattle in Japan, estimated breeding values for 305-day milk yield have been published since 1989, and those for lactation persistency have been published since 2008 (Hagiya, 2019). When genetic evaluation was initiated, selection focused on total yields during lactation, such as milk and fat yields. Later, lactation persistency was included as a selection trait. Both selected traits (i.e., higher 305-day milk yield and higher lactation persistency) prolonged lactation persistency and delayed the peak yield day. The Wilmink exponential term is associated with early lactation through peak lactation (Wilmink, 1987). The later the peak day, the slower the increase in milk yield during early lactation, leading to a smaller absolute value for the optimal exponential term. As a result of the improvement in lactation yield and persistency, we can infer that the peak day has become later for all parities and thus the exponential term that optimally escribes the lactation curve has changed.

We then evaluated the mean residuals for each value of Wilmink's exponential term for the first through fifth parities in 2017 (Figures 5 through 9). For the first and second parities, whereas the value of –0.05 for Wilmink's exponential term yielded small residuals throughout lactation, those of –0.02

and –0.03 resulted in large residuals, especially in early lactation. Similarly, the residuals were smallest when the exponential value was –0.04 for the third parity and –0.03 for the fourth and fifth parities; in addition, the residuals during early lactation were larger when the exponential term was –0.07 in both of these cases. These results suggest that, when an optimized value for Wilmink's exponential term is used, the estimated lactation curves are accurate, especially for the period from just after calving to around 40 days afterward.

In conclusion, average daily milk yield among Holstein cows in Japan increased from the first through third parities and then plateaued for later parities. The shapes of the lactation curve showed that the peak day of lactation occurred progressively later each year, and milk yield increased over the entire lactation period. Milk yield particularly tended to increase from mid-lactation to the end of lactation. Lactation persistency increased with test year for all parities and was highest for first parity. Here, we assessed various values for Wilmink's exponential term to achieve accurate lactation curves. For the Wilmink exponential term, the currently adopted value (–0.05) is appropriate for the first and second parities but should be revised for each later parity. In the future, the shape of the lactation curve likely will change because the peak day is expected to be later and milk yield is anticipated to increase. Therefore, the lactation curve model should be reevaluated periodically. We recommend incorporating an optimized value that accounts for calving year, parity, and the population of interest when using Wilmink’s function to estimate a lactation curve.

**CONFLICT OF INTERESTS**

The authors declare no conflict of interest.

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Legends

Figure 1. Trends in milk yield for each parity (first through fifth) according to test year.

Figure 2. Average daily milk yield for (a) first parity, (b) second parity, and (c) third parity.

Figure 3. Trends of peak milk-yield day for each parity.

Figure 4. Trends of lactation persistency for each parity, where  and  are test-day milk yields at 60 and 240 days in milk, respectively.

Figure 5. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for first parity in 2017.

Figure 6. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for second parity in 2017.

Figure 7. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for third parity in 2017.

Figure 8. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for fourth parity in 2017.

Figure 9. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for fifth parity in 2017.

**Table 1**. Number of records and mean, standard deviation (SD), minimum, and maximum milk yield (in kilograms) for each parity

| | | Milk yield (kg) | | | |
|---|---|---|---|---|---|
| Parity | No. of records | Mean | SD | Minimum | Maximum |
| 1 | 32,747,760 | 26.1 | 6.3 | 0.1 | 98.0 |
| 2 | 27,170,813 | 30.3 | 8.6 | 0.1 | 107.4 |
| 3 | 19,959,961 | 31.5 | 9.3 | 0.2 | 107.0 |
| 4 | 13,204,051 | 31.4 | 9.5 | 0.1 | 110.3 |
| 5 | 7,889,213 | 30.8 | 9.5 | 0.2 | 99.6 |

**Table 2.** Optimal values for Wilmink's exponential term according to parity

| | Parity | | | | |
|---|---|---|---|---|---|
| Year | 1 | 2 | 3 | 4 | 5 |
| 1991 | –0.06 | –0.05 | –0.04 | –0.04 | –0.04 |
| 1992 | –0.07 | –0.06 | –0.05 | –0.05 | –0.04 |
| 1993 | –0.06 | –0.05 | –0.04 | –0.04 | –0.04 |
| 1994 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 1995 | –0.05 | –0.06 | –0.04 | –0.04 | –0.04 |
| 1996 | –0.06 | –0.05 | –0.04 | –0.04 | –0.03 |
| 1997 | –0.05 | –0.05 | –0.04 | –0.04 | –0.04 |
| 1998 | –0.06 | –0.05 | –0.04 | –0.04 | –0.03 |
| 1999 | –0.06 | –0.05 | –0.04 | –0.04 | –0.03 |
| 2000 | –0.05 | –0.05 | –0.04 | –0.04 | –0.03 |
| 2001 | –0.05 | –0.06 | –0.04 | –0.04 | –0.03 |
| 2002 | –0.05 | –0.05 | –0.04 | –0.04 | –0.04 |
| 2003 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2004 | –0.04 | –0.04 | –0.04 | –0.04 | –0.03 |
| 2005 | –0.05 | –0.05 | –0.04 | –0.03 | –0.04 |
| 2006 | –0.05 | –0.04 | –0.03 | –0.03 | –0.03 |
| 2007 | –0.05 | –0.05 | –0.04 | –0.03 | –0.04 |
| 2008 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2009 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2010 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2011 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2012 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2013 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2014 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2015 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2016 | –0.05 | –0.05 | –0.04 | –0.03 | –0.04 |
| 2017 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |
| 2018 | –0.05 | –0.05 | –0.04 | –0.03 | –0.03 |

**Appendix 1**. Root mean squared error for each value of Wilmink's exponential term for first parity

| | Exponential term | | | | | |
|---|---|---|---|---|---|---|
| Year | –0.02 | –0.03 | –0.04 | –0.05 | –0.06 | –0.07 |
| 1991 | 4.19978 | 4.19887 | 4.19826 | 4.19800 | **4.19788** | 4.19797 |
| 1992 | 4.28845 | 4.28750 | 4.28683 | 4.28654 | 4.28639 | **4.28635** |
| 1993 | 4.35740 | 4.35646 | 4.35583 | 4.35552 | **4.35535** | 4.35544 |
| 1994 | 4.27980 | 4.27893 | 4.27832 | **4.27810** | 4.27812 | 4.27815 |
| 1995 | 4.46187 | 4.46100 | 4.46048 | **4.46027** | 4.46033 | 4.46046 |
| 1996 | 4.51796 | 4.51699 | 4.51640 | 4.51608 | **4.51605** | 4.51607 |
| 1997 | 4.52994 | 4.52916 | 4.52866 | **4.52839** | 4.52839 | 4.52856 |
| 1998 | 4.56493 | 4.56409 | 4.56357 | 4.56334 | **4.56332** | 4.56347 |
| 1999 | 4.55989 | 4.55898 | 4.55838 | 4.55813 | **4.55809** | 4.55829 |
| 2000 | 4.66763 | 4.66688 | 4.66646 | **4.66631** | 4.66639 | 4.66662 |
| 2001 | 4.72555 | 4.72467 | 4.72425 | **4.72408** | 4.72410 | 4.72424 |
| 2002 | 4.79029 | 4.78945 | 4.78894 | **4.78873** | 4.78877 | 4.78898 |
| 2003 | 4.85207 | 4.85127 | 4.85092 | **4.85081** | 4.85085 | 4.85112 |
| 2004 | 4.88208 | 4.88151 | **4.88115** | 4.88122 | 4.88145 | 4.88181 |
| 2005 | 4.90162 | 4.90091 | 4.90055 | **4.90052** | 4.90062 | 4.90087 |
| 2006 | 4.85524 | 4.85459 | 4.85429 | **4.85419** | 4.85434 | 4.85457 |
| 2007 | 4.82695 | 4.82630 | 4.82595 | **4.82587** | 4.82601 | 4.82628 |
| 2008 | 4.84001 | 4.83934 | 4.83903 | **4.83900** | 4.83905 | 4.83934 |
| 2009 | 4.94579 | 4.94518 | 4.94480 | **4.94474** | 4.94486 | 4.94513 |
| 2010 | 5.01000 | 5.00929 | 5.00892 | **5.00881** | 5.00889 | 5.00912 |
| 2011 | 5.04603 | 5.04523 | 5.04484 | **5.04471** | 5.04483 | 5.04509 |
| 2012 | 5.08316 | 5.08245 | 5.08211 | **5.08197** | 5.08206 | 5.08220 |
| 2013 | 5.11878 | 5.11804 | 5.11767 | **5.11759** | 5.11765 | 5.11786 |
| 2014 | 5.06484 | 5.06409 | 5.06369 | **5.06353** | 5.06357 | 5.06375 |
| 2015 | 5.11618 | 5.11542 | 5.11504 | **5.11496** | 5.11502 | 5.11526 |
| 2016 | 5.19266 | 5.19200 | 5.19166 | **5.19159** | 5.19169 | 5.19190 |
| 2017 | 5.22223 | 5.22151 | 5.22113 | **5.22103** | 5.22107 | 5.22126 |
| 2018 | 5.29739 | 5.29670 | 5.29625 | **5.29610** | 5.29615 | 5.29638 |

Bold text indicates the lowest root mean squared error for each year.

**Appendix 2.** Root mean squared error for each value of Wilmink's exponential term for second parity

| | Exponential term | | | | | |
|---|---|---|---|---|---|---|
| Year | –0.02 | –0.03 | –0.04 | –0.05 | –0.06 | –0.07 |
| 1991 | 5.03517 | 5.03397 | 5.03330 | **5.03303** | 5.03306 | 5.03335 |
| 1992 | 5.15553 | 5.15404 | 5.15323 | 5.15287 | **5.15286** | 5.15305 |
| 1993 | 5.22440 | 5.22302 | 5.22216 | **5.22182** | 5.22184 | 5.22213 |
| 1994 | 5.16027 | 5.15894 | 5.15819 | **5.15794** | 5.15800 | 5.15821 |
| 1995 | 5.36566 | 5.36416 | 5.36336 | 5.36298 | **5.36295** | 5.36314 |
| 1996 | 5.46153 | 5.46014 | 5.45937 | **5.45911** | 5.45916 | 5.45953 |
| 1997 | 5.42180 | 5.42034 | 5.41967 | **5.41940** | 5.41950 | 5.41984 |
| 1998 | 5.47614 | 5.47472 | 5.47391 | **5.47361** | 5.47366 | 5.47398 |
| 1999 | 5.50994 | 5.50836 | 5.50746 | **5.50714** | 5.50728 | 5.50768 |
| 2000 | 5.65789 | 5.65654 | 5.65577 | **5.65550** | 5.65551 | 5.65578 |
| 2001 | 5.70216 | 5.70067 | 5.69973 | 5.69944 | **5.69941** | 5.69970 |
| 2002 | 5.76143 | 5.75997 | 5.75927 | **5.75908** | 5.75923 | 5.75960 |
| 2003 | 5.85785 | 5.85650 | 5.85575 | **5.85563** | 5.85570 | 5.85624 |
| 2004 | 5.90636 | 5.90526 | **5.90492** | 5.90495 | 5.90530 | 5.90582 |
| 2005 | 5.95901 | 5.95791 | 5.95730 | **5.95713** | 5.95735 | 5.95776 |
| 2006 | 5.86813 | 5.86726 | **5.86686** | 5.86693 | 5.86730 | 5.86791 |
| 2007 | 5.85167 | 5.85063 | 5.85021 | **5.85013** | 5.85026 | 5.85074 |
| 2008 | 5.86968 | 5.86849 | 5.86798 | **5.86782** | 5.86805 | 5.86861 |
| 2009 | 5.93186 | 5.93092 | 5.93036 | **5.93034** | 5.93064 | 5.93113 |
| 2010 | 6.00902 | 6.00786 | 6.00739 | **6.00732** | 6.00761 | 6.00814 |
| 2011 | 6.13910 | 6.13797 | 6.13741 | **6.13728** | 6.13747 | 6.13799 |
| 2012 | 6.20152 | 6.20033 | 6.19973 | **6.19950** | 6.19968 | 6.20022 |
| 2013 | 6.22706 | 6.22581 | 6.22523 | **6.22502** | 6.22527 | 6.22573 |
| 2014 | 6.19727 | 6.19610 | 6.19557 | **6.19542** | 6.19566 | 6.19600 |
| 2015 | 6.22293 | 6.22171 | 6.22102 | **6.22094** | 6.22108 | 6.22149 |
| 2016 | 6.28048 | 6.27935 | 6.27880 | **6.27880** | 6.27905 | 6.27948 |
| 2017 | 6.34843 | 6.34739 | 6.34694 | **6.34680** | 6.34700 | 6.34759 |
| 2018 | 6.44157 | 6.44038 | 6.43967 | **6.43946** | 6.43949 | 6.43975 |

Bold text indicates the lowest root mean squared error for each year.

**Appendix 3.** Root mean squared error for each value of Wilmink's exponential term for third parity

| | Exponential term | | | | | |
|---|---|---|---|---|---|---|
| Year | –0.02 | –0.03 | –0.04 | –0.05 | –0.06 | –0.07 |
| 1991 | 5.35429 | 5.35338 | **5.35308** | 5.35309 | 5.35341 | 5.35390 |
| 1992 | 5.45507 | 5.45397 | 5.45344 | **5.45337** | 5.45356 | 5.45402 |
| 1993 | 5.49624 | 5.49532 | **5.49503** | 5.49526 | 5.49574 | 5.49641 |
| 1994 | 5.43606 | 5.43533 | **5.43507** | 5.43528 | 5.43578 | 5.43649 |
| 1995 | 5.68128 | 5.68030 | **5.67998** | 5.68002 | 5.68039 | 5.68107 |
| 1996 | 5.75300 | 5.75227 | **5.75200** | 5.75230 | 5.75280 | 5.75343 |
| 1997 | 5.73672 | 5.73599 | **5.73570** | 5.73591 | 5.73651 | 5.73714 |
| 1998 | 5.80170 | 5.80063 | **5.80022** | 5.80028 | 5.80068 | 5.80126 |
| 1999 | 5.86100 | 5.85979 | **5.85936** | 5.85941 | 5.85983 | 5.86038 |
| 2000 | 6.02368 | 6.02285 | **6.02251** | 6.02270 | 6.02311 | 6.02373 |
| 2001 | 6.05976 | 6.05883 | **6.05856** | 6.05878 | 6.05921 | 6.05986 |
| 2002 | 6.11131 | 6.11030 | **6.10991** | 6.11000 | 6.11046 | 6.11117 |
| 2003 | 6.23317 | 6.23228 | **6.23198** | 6.23227 | 6.23278 | 6.23359 |
| 2004 | 6.30601 | 6.30528 | **6.30506** | 6.30537 | 6.30593 | 6.30671 |
| 2005 | 6.31639 | 6.31566 | **6.31548** | 6.31585 | 6.31634 | 6.31708 |
| 2006 | 6.26881 | **6.26830** | 6.26836 | 6.26874 | 6.26937 | 6.27016 |
| 2007 | 6.23819 | 6.23759 | **6.23741** | 6.23773 | 6.23826 | 6.23896 |
| 2008 | 6.25431 | 6.25380 | **6.25373** | 6.25413 | 6.25459 | 6.25529 |
| 2009 | 6.32948 | 6.32904 | **6.32903** | 6.32944 | 6.33003 | 6.33081 |
| 2010 | 6.39029 | 6.38965 | **6.38954** | 6.38973 | 6.39020 | 6.39088 |
| 2011 | 6.49432 | 6.49365 | **6.49365** | 6.49390 | 6.49431 | 6.49494 |
| 2012 | 6.58762 | 6.58695 | **6.58666** | 6.58687 | 6.58737 | 6.58798 |
| 2013 | 6.67892 | 6.67827 | **6.67825** | 6.67865 | 6.67916 | 6.67995 |
| 2014 | 6.62507 | 6.62453 | **6.62447** | 6.62475 | 6.62533 | 6.62603 |
| 2015 | 6.68724 | 6.68664 | **6.68655** | 6.68695 | 6.68746 | 6.68813 |
| 2016 | 6.76462 | 6.76409 | **6.76404** | 6.76435 | 6.76481 | 6.76552 |
| 2017 | 6.76600 | 6.76541 | **6.76539** | 6.76563 | 6.76631 | 6.76699 |
| 2018 | 6.85390 | 6.85333 | **6.85327** | 6.85350 | 6.85396 | 6.85460 |

Bold text indicates the lowest root mean squared error for each year.

**Appendix 4.** Root mean squared error for each value of Wilmink's exponential term for fourth parity

| | Exponential term | | | | | |
|---|---|---|---|---|---|---|
| Year | –0.02 | –0.03 | –0.04 | –0.05 | –0.06 | –0.07 |
| 1991 | 5.50977 | 5.50910 | **5.50893** | 5.50915 | 5.50953 | 5.51012 |
| 1992 | 5.62859 | 5.62774 | 5.62742 | **5.62758** | 5.62804 | 5.62861 |
| 1993 | 5.67818 | 5.67741 | **5.67736** | 5.67761 | 5.67813 | 5.67881 |
| 1994 | 5.55794 | **5.55747** | 5.55748 | 5.55786 | 5.55852 | 5.55928 |
| 1995 | 5.73658 | 5.73585 | **5.73564** | 5.73589 | 5.73644 | 5.73706 |
| 1996 | 5.85180 | 5.85106 | **5.85093** | 5.85133 | 5.85189 | 5.85270 |
| 1997 | 5.83601 | 5.83540 | **5.83532** | 5.83565 | 5.83631 | 5.83708 |
| 1998 | 5.95045 | 5.94995 | **5.94993** | 5.95033 | 5.95108 | 5.95197 |
| 1999 | 6.01302 | 6.01226 | **6.01214** | 6.01258 | 6.01315 | 6.01399 |
| 2000 | 6.18527 | 6.18456 | **6.18447** | 6.18478 | 6.18543 | 6.18624 |
| 2001 | 6.21026 | 6.20945 | **6.20937** | 6.20976 | 6.21037 | 6.21123 |
| 2002 | 6.30004 | 6.29943 | **6.29939** | 6.29992 | 6.30067 | 6.30154 |
| 2003 | 6.34897 | **6.34841** | 6.34844 | 6.34889 | 6.34954 | 6.35049 |
| 2004 | 6.43263 | 6.43206 | **6.43203** | 6.43250 | 6.43313 | 6.43402 |
| 2005 | 6.45591 | **6.45542** | 6.45544 | 6.45585 | 6.45641 | 6.45729 |
| 2006 | 6.39423 | **6.39378** | 6.39380 | 6.39423 | 6.39483 | 6.39556 |
| 2007 | 6.37057 | **6.37017** | 6.37021 | 6.37067 | 6.37139 | 6.37215 |
| 2008 | 6.42906 | **6.42876** | 6.42887 | 6.42932 | 6.43002 | 6.43076 |
| 2009 | 6.50343 | **6.50323** | 6.50351 | 6.50393 | 6.50472 | 6.50554 |
| 2010 | 6.57547 | **6.57513** | 6.57531 | 6.57579 | 6.57644 | 6.57725 |
| 2011 | 6.63677 | **6.63629** | 6.63635 | 6.63660 | 6.63732 | 6.63804 |
| 2012 | 6.73815 | **6.73775** | 6.73787 | 6.73830 | 6.73891 | 6.73967 |
| 2013 | 6.83109 | **6.83067** | 6.83067 | 6.83107 | 6.83167 | 6.83230 |
| 2014 | 6.79379 | **6.79347** | 6.79366 | 6.79415 | 6.79487 | 6.79572 |
| 2015 | 6.87253 | **6.87224** | 6.87236 | 6.87284 | 6.87357 | 6.87434 |
| 2016 | 6.94857 | **6.94809** | 6.94812 | 6.94848 | 6.94909 | 6.94978 |
| 2017 | 6.96206 | **6.96170** | 6.96175 | 6.96217 | 6.96271 | 6.96337 |
| 2018 | 7.01739 | **7.01713** | 7.01727 | 7.01768 | 7.01831 | 7.01890 |

Bold text indicates the lowest root mean squared error for each year.

**Appendix 5.** Root mean squared error for each value of Wilmink's exponential term for fifth parity

| | Exponential term | | | | | |
|---|---|---|---|---|---|---|
| Year | –0.02 | –0.03 | –0.04 | –0.05 | –0.06 | –0.07 |
| 1991 | 5.51598 | 5.51536 | **5.51524** | 5.51550 | 5.51599 | 5.51664 |
| 1992 | 5.64686 | 5.64629 | **5.64626** | 5.64661 | 5.64718 | 5.64795 |
| 1993 | 5.69943 | 5.69889 | **5.69881** | 5.69916 | 5.69980 | 5.70060 |
| 1994 | 5.63557 | **5.63518** | 5.63533 | 5.63580 | 5.63645 | 5.63728 |
| 1995 | 5.79484 | 5.79428 | **5.79422** | 5.79453 | 5.79512 | 5.79583 |
| 1996 | 5.84025 | **5.83981** | 5.83999 | 5.84045 | 5.84130 | 5.84219 |
| 1997 | 5.84997 | 5.84941 | **5.84934** | 5.84968 | 5.85028 | 5.85108 |
| 1998 | 5.93922 | **5.93881** | 5.93899 | 5.93943 | 5.94011 | 5.94090 |
| 1999 | 6.00883 | **6.00838** | 6.00855 | 6.00906 | 6.00978 | 6.01071 |
| 2000 | 6.14508 | **6.14468** | 6.14485 | 6.14538 | 6.14609 | 6.14697 |
| 2001 | 6.19826 | **6.19767** | 6.19779 | 6.19828 | 6.19905 | 6.19999 |
| 2002 | 6.30239 | 6.30174 | **6.30166** | 6.30209 | 6.30271 | 6.30356 |
| 2003 | 6.39353 | **6.39299** | 6.39305 | 6.39347 | 6.39424 | 6.39504 |
| 2004 | 6.46029 | **6.45998** | 6.46015 | 6.46062 | 6.46132 | 6.46212 |
| 2005 | 6.44679 | 6.44619 | **6.44614** | 6.44650 | 6.44707 | 6.44778 |
| 2006 | 6.39065 | **6.39038** | 6.39058 | 6.39117 | 6.39195 | 6.39291 |
| 2007 | 6.36683 | 6.36635 | **6.36624** | 6.36660 | 6.36711 | 6.36774 |
| 2008 | 6.40173 | **6.40147** | 6.40167 | 6.40212 | 6.40281 | 6.40354 |
| 2009 | 6.49743 | **6.49719** | 6.49741 | 6.49788 | 6.49862 | 6.49940 |
| 2010 | 6.55455 | **6.55422** | 6.55436 | 6.55481 | 6.55547 | 6.55619 |
| 2011 | 6.62589 | **6.62566** | 6.62586 | 6.62631 | 6.62690 | 6.62760 |
| 2012 | 6.70965 | **6.70919** | 6.70928 | 6.70969 | 6.71030 | 6.71102 |
| 2013 | 6.77053 | **6.77017** | 6.77034 | 6.77067 | 6.77125 | 6.77191 |
| 2014 | 6.73367 | **6.73330** | 6.73340 | 6.73386 | 6.73452 | 6.73525 |
| 2015 | 6.85033 | **6.85016** | 6.85039 | 6.85094 | 6.85164 | 6.85247 |
| 2016 | 6.97264 | 6.97225 | **6.97223** | 6.97261 | 6.97306 | 6.97371 |
| 2017 | 7.00409 | **7.00386** | 7.00401 | 7.00452 | 7.00513 | 7.00590 |
| 2018 | 7.06315 | **7.06301** | 7.06327 | 7.06372 | 7.06434 | 7.06503 |

Bold text indicates the lowest root mean squared error for each year.

Figure1

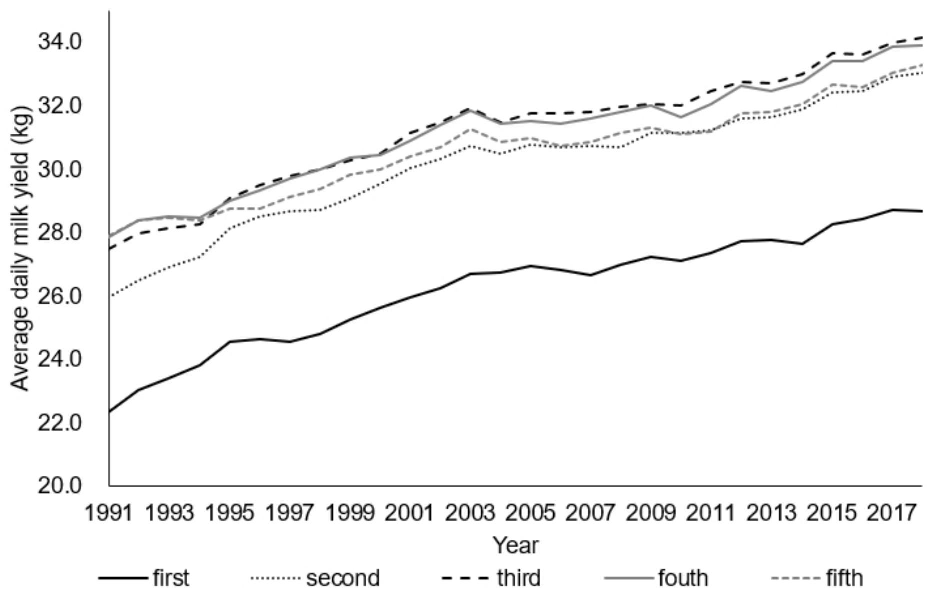


Figure 1. Trends in milk yield for each parity (first through fifth) according to test year.

511x374mm (38 x 38 DPI)

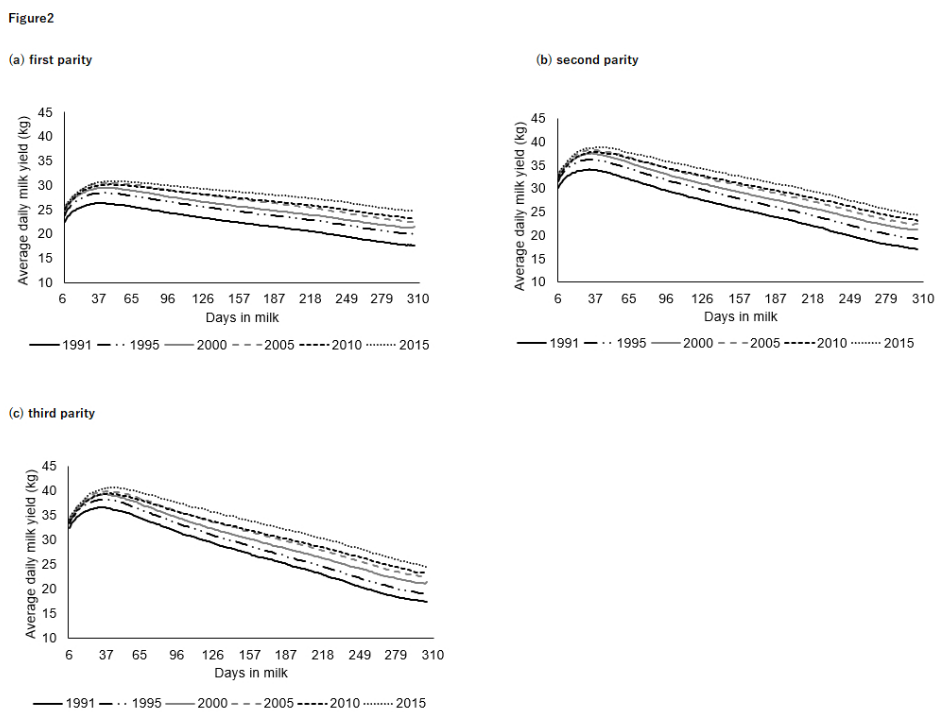


Figure 2. Average daily milk yield for (a) first parity, (b) second parity, and (c) third parity.

471x335mm (57 x 57 DPI)

Figure3

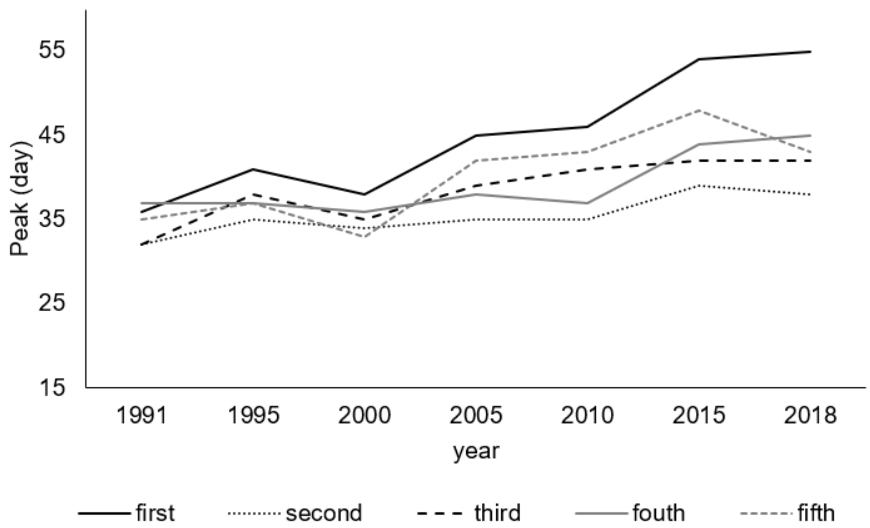


Figure 3. Trends of peak milk-yield day for each parity.

534x381mm (38 x 38 DPI)

Figure4

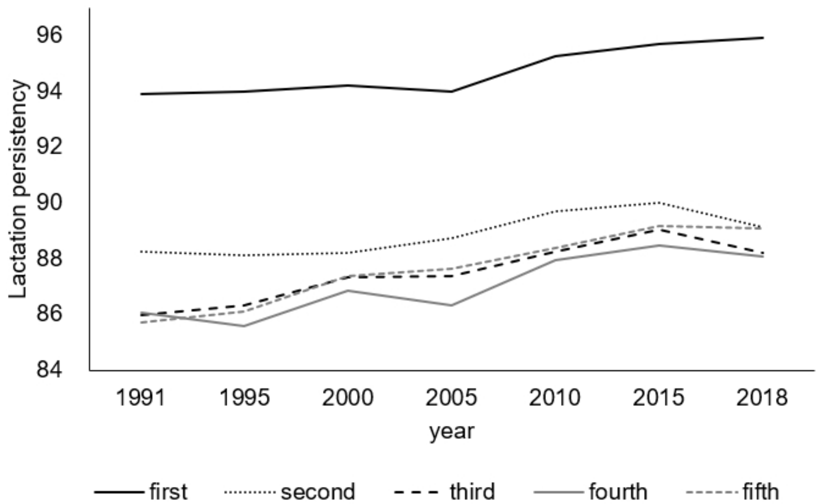


Figure 4. Trends of lactation persistency for each parity, where and are test-day milk yields at 60 and 240 days in milk, respectively.

547x402mm (38 x 38 DPI)

Figure5

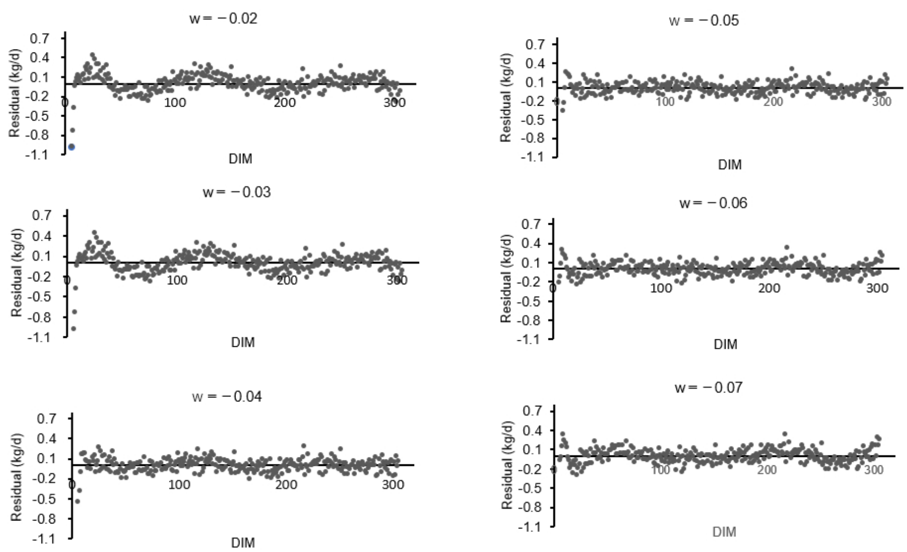


Figure 5. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for first parity in 2017.

673x435mm (38 x 38 DPI)

Figure6

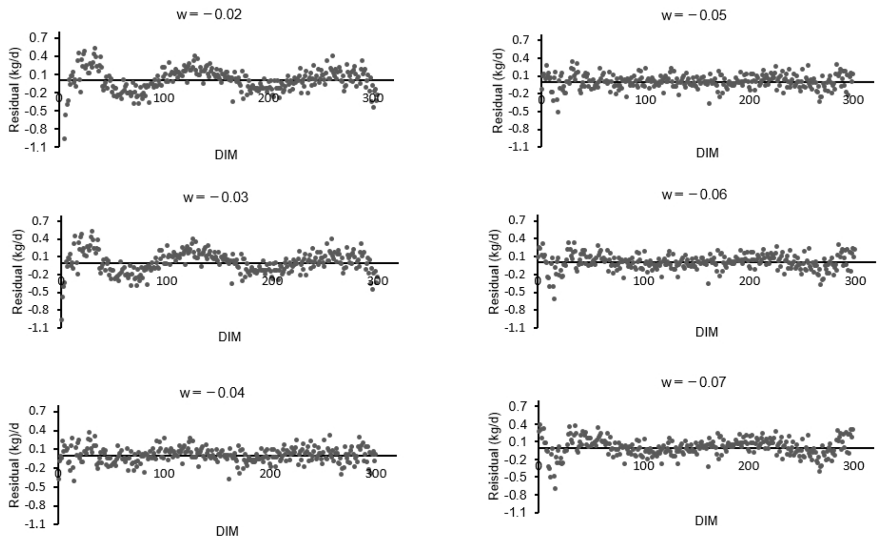


Figure 6. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for second parity in 2017.

702x441mm (38 x 38 DPI)

Figure7

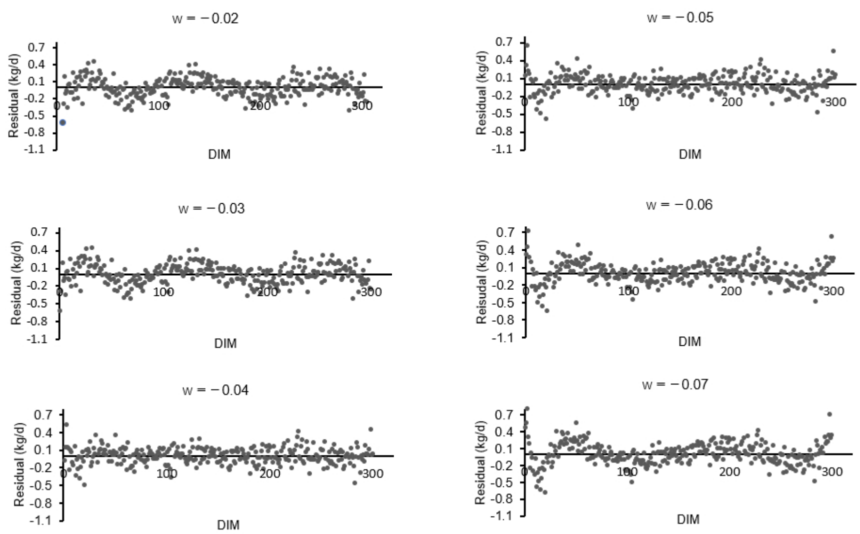


Figure 7. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for third parity in 2017.

709x441mm (38 x 38 DPI)

Figure8

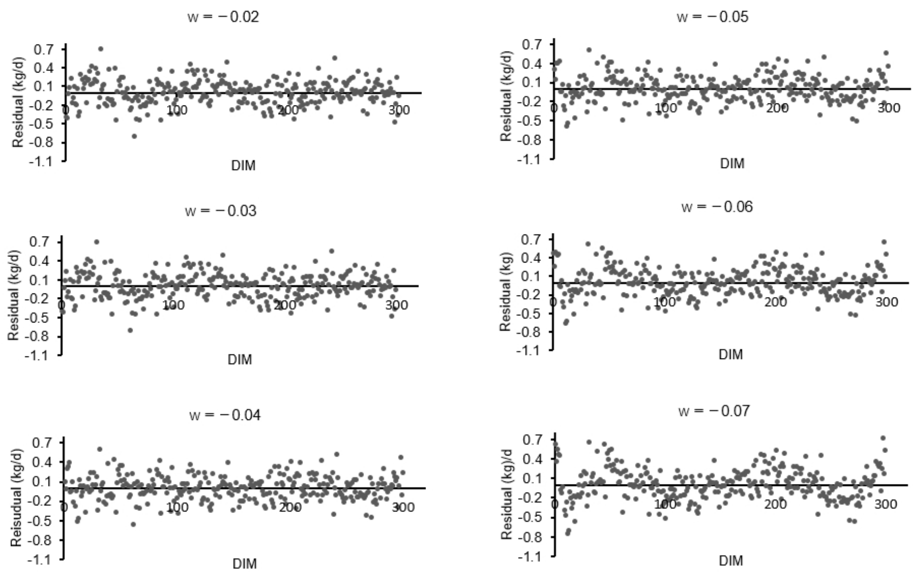


Figure 8. Mean residuals on days in milk (DIM) for each value of Wilmink’s exponential term for fourth parity in 2017.

657x437mm (38 x 38 DPI)

Figure9

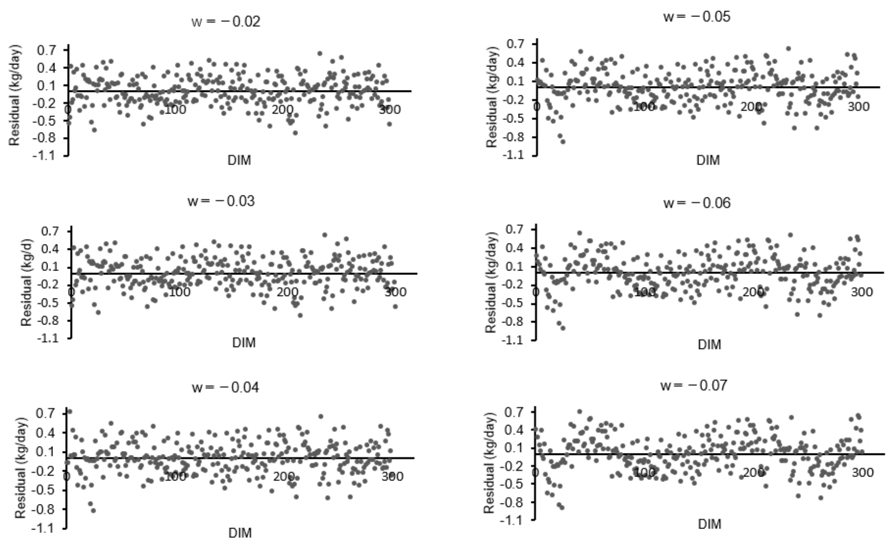


Figure 9. Mean residuals on days in milk (DIM) for each value of Wilmink's exponential term for fifth parity in 2017.

681x440mm (38 x 38 DPI)