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Four mathematical models (compartmental (C), modified compartmental (MC), Wood (W), and Adams-Bell (AB)) were fitted to different patterns of egg production data in laying hens to compare the fits of these models.

The egg production data were obtained from the two lines which were founded on a basis of divergent selection for egg quality traits. Egg production data of individual hens were classified in six patterns. The average hen-day rates of lay were calculated for every 10 days from the first egg for each pattern. The comparison of fits was based on the R^2 adjusted for degrees of freedom and Akaike's information criterion (AIC).

The fits of C, MC and AB models were similar for the egg production data showing a general pattern of curve. However, the model parameters could not be estimated by these three models for the data whose patterns showed abrupt decreases after the peak of egg production. The data in which the period from the first egg to the peak of egg production was short and linearly decreased after reaching the peak showed poor fits of the models. The parameters of all patterns could be estimated by the W model, but the fits were lower than those of the others.

Introduction

Egg production in chickens is generally presented as an egg production curve using a non-linear mathematical model based on the records summarized on a weekly or a monthly basis for a group of hens. Four non-linear equations have been known to be prominent in mathematical modeling of egg production in poultry, namely : 1) Wood model, originally applied to lactation curve in dairy cattle (WOOD, 1967), 2) compartmental model, originally applied to egg production in the drosophila (McMILLAN et al., 1970 a, b), 3) modified compartmental model that was a combined logistic model (YANG et al., 1989), and 4) Adams-Bell model that was improved using a non linear model for growth curve of chickens (ADAMS and BELL, 1980). These models have been compared on the basis of goodness of fit or predictability (GAVORA et al., 1982 ; McMILLAN et al., 1986 ; CASON and BRITTON, 1988). However, there were large differences on the fit and the predictability for the egg production records used in each model (GAVORA et al., 1971). The best is not be easily determined among these models, because various patterns of egg production curve may possibly exist.

The aim of this study is to inspect the patterns of egg production curve, and then to examine the applicability of the above models to different patterns of egg production.

Materials and Methods

The egg production data used in this study were obtained from 1,113 hens of the 18 th~23 th generation of a White Leghorn population of two selected lines (MITSUMOTO and MIYOSHI, 1989). These lines were founded on a basis of divergent selection for high and low yolk-albumen ratio in 12 generations since 1970, and relaxed selection from generation 13. Hens of both lines were hatched from June to July and reared under the regimes of feeding and management maintaining almost the same in every generation. Egg production data were recorded from November of the hatching year to December of the next year. The data were recorded daily for each hen from the first egg, and then summarized as 10-day egg numbers. The shapes of egg production records of every hen were inspected, and six common patterns were derived as shown in Fig. 1. The hens that had a similar egg production pattern were classified into one of them, and the hens with record of the irregular pattern (out of these patterns), died or eliminated in the main production period were excluded from the analyses.

The day of first egg was defined as $t = 0$, and the average hen-day rates of lay of each pattern were calculated for every 10 days. The data were fitted to the four non-linear models as follows :

1. compartmental (C) (GAVORA *et al.*, 1971)

   $Y_t = a (e^{-bt}) [1 - e^{-c(t-d)}]$

2. modified compartmental (MC) (YANG *et al.*, 1989)

   $Y_t = a (e^{-bt}) / [1 + e^{-c(t-d)}]$

3. Wood (W) (WOOD, 1967)

   $Y_t = f (t^g) (e^{-ht})$

4. Adams-Bell (AB) (ADAMS and BELL, 1980)

   $Y_t = 1 / [0.01 + mr^{(t-n)}] - p (t-q)$

where :

Y_t, the average hen-day rate of lay at day t ;

e, the base of natural logarithm, and ;

$a, b, c, d, f, g, h, m, n, p, q$, the parameters to be estimated.

The day t was the middle point of hen-day egg production over 10 days. For example, hen-day egg production at $t = 25$ means the rate of lay in a period from 21th to 30th day after the first egg.

The comparison of fit of each model depended on the R^2 adjusted for degrees of freedom, and Akaike's information criterion (AIC). The NLIN procedure of SAS (SAS Institute, 1985) was used to estimate the parameters in each model.

Results and Discussion

Egg production of individual hens showed various patterns of curves. The most distinguishable six patterns were achieved and shown in Fig. 1. It indicated that hens
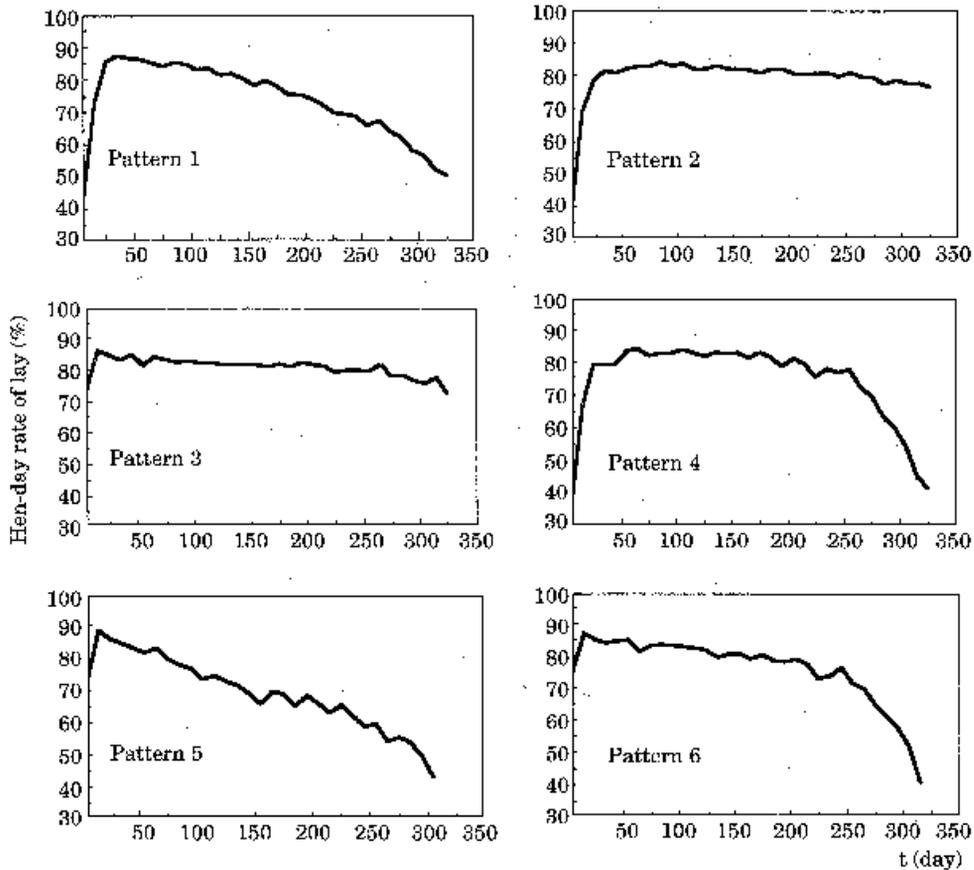
Fig. 1. Patterns of egg production curve.

in patterns 1 and 2 got the peak of egg production at about 1~2 months after onset of lay, and then gradually decreased. Patterns 4 and 6 showed abrupt decreases at the end of egg production period. A linear decrease after the peak was observed in pattern 5. A total of 725 hens (65%) whose shape of egg production curves were distinguishable. Of them the highest frequency was found in pattern 2 having 194 hens, followed by pattern 3 with 145 and pattern 1 with 130 hens (Table 1). The number of hens whose laying curves were not classifiable or irregular patterns was 388 out of 1,113 (35%). This indicated the existence of a considerable number of irregular patterns in egg production of laying hens. Moreover, the hens even had a normal shape in egg production, the convergence for records of these individual hens was seldomly met. Therefore, the average hen day rates of lay were firstly calculated for each pattern and then fitted to the models.

Table 1 presents the parameter estimates, the coefficients of determination (R^2) and Akaike's information criterion (AIC) for each pattern in the four models. The parameters of all patterns could be estimated by the W model, but R^2 was almost the same (> 0.99) and AIC was higher than those of the other models. These results agree with the reports of McMILLAN et al. (1986) and YANG et al. (1989) that Wood model does not show

Table 1. Parameter estimates, coefficients of determination (R^2) and Akaike's information criterion (AIC) of four models depending on six egg production patterns.

Model[a]	Parameter	Patterns of egg production curve (Number of hens)					
		1 (130)	2 (194)	3 (145)	4 (105)	5 (56)	6 (95)
C	a	99.606	85.002	85.753		91.536	
		(1.962)	(.434)	(.552)		(1.374)	
	b	.00166	.00028	.00035		.00187	
		(.00010)	(.00002)	(.00003)		(.00009)	
	c	.07294	.10475	1.3566		.34988	
		(.01071)	(.00549)	(.0896)		(.58143)	
	d	−3.4086	.91911	3.5136		.32191	
		(1.5989)	(.39740)	(0)		(7.7295)	
	R^2	.998	.999	.999		.998	
	AIC	38.61	3.82	17.18		33.64	
MC	a	98.042	84.530	85.753		91.531	
		(1.788)	(.470)	(.552)		(1.371)	
	b	.00159	.00025	.00035		.00187	
		(.00019)	(.00003)	(.00003)		(.00009)	
	c	.12030	.13857	1.3955		.37576	
		(.01763)	(.00889)	(0)		(.6122)	
	d	6.0493	5.828	3.6575		1.2193	
		(1.0257)	(.302)	(.1101)		(6.1204)	
	R^2	.998	.999	.999		.998	
	AIC	40.02	7.99	17.18		33.64	
W	f	35.587	37.492	72.792	25.034	68.818	61.037
		(2.670)	(3.112)	(2.322)	(3.826)	(3.866)	(6.175)
	g	.26819	.21234	.04431	.35324	.08060	.12028
		(.0213)	(.02302)	(.0092)	(.0424)	(.01704)	(.02972)
	h	.00349	.00169	.00067	.00378	.00255	.00228
		(.00021)	(.00021)	(.00009)	(.00038)	(.00019)	(.00032)
	R^2	.998	.998	.999	.993	.998	.994
	AIC	38.83	41.30	21.48	55.61	34.05	51.38
AB	m	.03625	.03626				
		(.00797)	(.00365)				
	n	−5.1214	−5.1299				
		(0)	(0)				
	p	.12085	.02101				
		(.00593)	(.00219)				
	q	−32.596	731.54				
		(11.421)	(96.07)				
	r	.88136	.86360				
		(.00062)	(.00033)				
	R^2	.998	.999				
	AIC	36.58	8.78				

Note : The empty cells are those in which converge was not met.
[a] C : compartmental ; MC : modified compartmental ; W : Wood ; AB : Adams-Bell.
The numbers in parentheses are standard errors of the estimated parameters.
R^2 : Adjusted for degrees of freedom.

applicability to the egg production curve. The C and MC models were estimable for
patterns 1, 2, 3 and 5, of which patterns 1, 2 and 3 were considered as typical shapes of
commercial laying flocks. McMULLAN (1986) also reported that it was possible to predict
egg production by linear regression in patterns 3 and 5, where the period from the first
egg to the peak of egg production is short and linearly decreases after getting the peak.
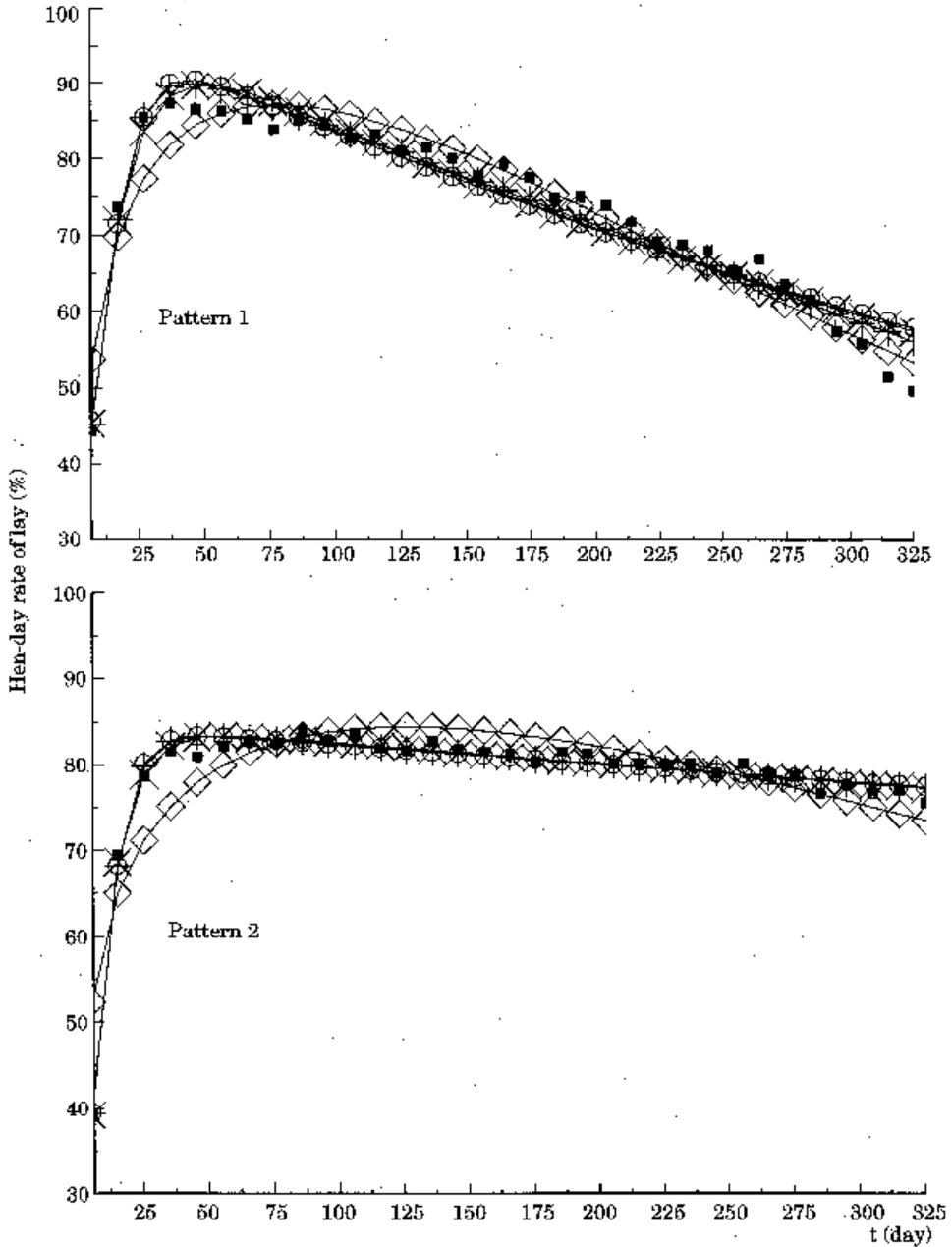The comparison on the basis of R^2 and AIC among the four models showed that the C

Fig. 2.   Fitting egg production data (■) of patterns 1 and 2 to compartmental (×),
modified compartmental (○), Wood (◇), Adams Bell (┃).

and MC models had better fits than the W model for patterns 1, 2, 3 and 5. However, no apparent difference was observed between the C and MC models, although YANG *et al.* (1989) reported that the MC model was superior to the C model. Parameters of the AB model were estimated only in patterns 1 and 2, and on the magnitude of R^2 and AIC, the fits of this model were similar to those of the C and MC models. These results are not consistent with the reports of CASON and BRITTON (1988) that R^2 of C model was significantly smaller than that of AB and the logistic models. The parameters of all models except for the W model were not estimated in patterns 4 and 6 with two phases of decrease, indicating that these models did not include the terms explaining for the second phase decrease of the shape. The plots between actual and predicted values by all models for patterns 1 and 2 are presented in Fig. 2.

Prediction of annual egg production is one of the aims of utilization of non-linear models. This study revealed the possible application of the non-linear models to the prediction of annual egg production for the large flocks. However, the prediction for the small groups of hens was difficult as indicated in the present study and the previous report of GAVORA *et al.* (1982) that the models showed differences in fit to different egg production patterns. From a view point of selection or breeding, the parameters and/or predicted values of the models should serve as the indices of egg production for individual hens, but it was impossible to apply the non-linear models for individuals because of the existence of the irregular patterns in egg production. Therefore, a better biological explanation can be attained if there is a suitable model that can predict the abilities of egg production for individual hens.

Acknowledgements

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鶏の産卵曲線に対する非線形モデルの適用

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鶏の産卵記録を種々の産卵様相に区分し，compartmental モデル（C），修正 compartmental モデル（MC），Wood モデル（W）および Adams Bell モデル（AB）の非線形モデルを当てはめ，その適合度を比較した。

産卵記録は卵構成を変化させる目的で選抜育種された鶏から得られた。それら個体の記録は大別して6種類の産卵パターンに分類された。各パターンについて，初産を揃えた10日毎の産卵率を産出し，モデル当てはめのデータとした。モデルの適合度の比較は自由度修正決定係数および赤池の情報量基準によった。

鶏の一般的な産卵様相を示すデータに対しては，C，MC および AB モデルが同等の適合度を示したが，産卵後期において急激な産卵率の低下を示すパターンに当てはめた場合には，収束解が得られなかった。また，短期間で産卵最盛期に達し，その後，直線的な推移を示すパターンに対するモデルの適合度は劣るものであった。W モデルは，分類した6種の産卵パターンに対し収束解が得られたが，その適合度は他のモデルより顕著に劣るものであった。

（家禽会誌, **33** : 178-184, 1996）

キーワード：産卵様相，非線形モデル，産卵鶏